Approximating a point using least-squares best-fitting polynomials

1. A noisy sensor is reading speed at a rate of once every five seconds, and the reading is in meters per second. The readings are as follows:

0, 0, 0, 0, –0.35, 1.84, 1.56, –1.12, –4.70, 2.95, 3.77, 1.97, 5.81, 8.11, 10.62, 11.88, 17.45

Use the five-point approximation shown in the course slides:

1. For best-fitting least-squares linear polynomials:

   $a_1 = -0.2y_{n-4} - 0.1y_{n-3} + 0.1y_{n-1} + 0.2y_n$

   $a_0 = -0.2y_{n-4} + 0.2y_{n-2} + 0.4y_{n-1} + 0.6y_n$

2. For best-fitting least-squares quadratic polynomials:

   $a_2 = (\ 2y_{n-4} - y_{n-3} - 2y_{n-2} - y_{n-1} + 2y_n)/14$

   $a_1 = (26y_{n-4} - 27y_{n-3} - 40y_{n-2} - 13y_{n-1} + 54y_n)/70$

   $a_0 = (\ 3y_{n-4} - 5y_{n-3} - 3y_{n-2} + 9y_{n-1} + 31y_n)/35$

These coefficients are found by explicitly calculating $(V^{\mathrm{T}}V)^{-1}V^{\mathrm{T}}$ and multiplying them by **y**. You do not have to memorize these coefficients: if needed, they would be given to you on an examination.

Starting with the fifth point, approximate the value at the point, at the point half a step into the past, and at the point one step into the future.

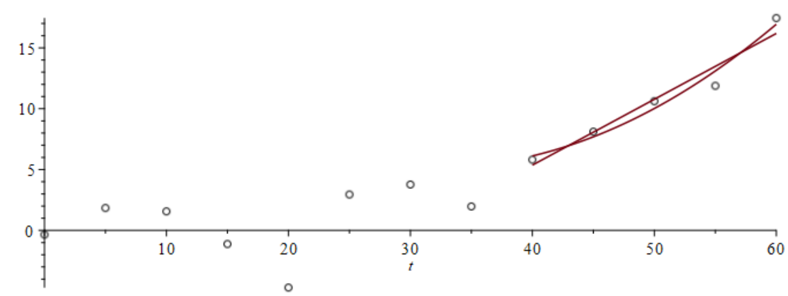
Answer: Starting with the 5th point, assuming all previous values are zero, and rounding the first two quadratic approximations to two points beyond the decimal, we have for $\delta = 0$, $\delta = -0.5$, and $\delta = 1.0$:

–0.21, 0.964, 1.602, 0.32, –2.886, –0.702, 2.19, 3.504, 5.968, 6.994, 10.024, 12.604, 16.184
–0.175, 0.7975, 1.354, 0.3365, –2.303, –0.5, 1.7655, 2.7715, 4.966, 6.376, 9.032, 11.3725, 14.8315
–0.28, 1.297, 2.098, 0.287, –4.052, –1.106, 3.039, 4.969, 7.972, 8.23, 12.008, 15.067, 18.889

–0.31, 1.54, 1.88, –0.70, –4.88, 1.44, 4.79, 3.04, 4.51, 8.22, 11.03, 11.90, 16.94
–0.19, 0.87, 1.39, 0.21, –2.55, –0.23, 2.09, 2.71, 4.78, 6.53, 9.16, 11.28, 14.93
–0.63, 3.312, 3.088, –3.278, –11.022, 6.374, 12.154, 3.334, 2.852, 12.53, 15.548, 12.592, 21.534

2. Plot the points with noise, and then plot the least-squares best-fitting polynomials that are used to estimate the the values at the last point assuming the first noisy signal was taken at time $t = 0$.

Answer:



3. With as much noise as was introduced into the data in Question 1, would it make more sense, or less sense, to use more points in finding the best-fitting least-squares polynomials?

Answer: The errors introduced into the data is quite significant, so more points would definitely give a much better approximation by eliminating some of that error.

Acknowledgements: Harshil Shaw and Aaran Muraleetharan for pointing out the word “linear” was repeated when the second use should have been “quadratic” in Question 1.